# Threshold-based synchrophasor monitoring of multiple outages on a detailed WECC system using area angle 

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# Threshold-based synchrophasor monitoring of multiple outages on a detailed WECC system using area angle 

## by

## Vikram Kumar Reddy Chiluka

A thesis submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of
MASTER OF SCIENCE

Major: Electrical Engineering

Program of Study Committee:
Ian Dobson, Major Professor
Venkataramana Ajjarapu
Manimaran Govindarasu

The student author, whose presentation of the scholarship herein was approved by the program of study committee, is solely responsible for the content of this thesis. The Graduate College will ensure this thesis is globally accessible and will not permit alterations after a degree is conferred.

Iowa State University

Ames, Iowa

## TABLE OF CONTENTS

## Page

LIST OF TABLES ..... iv
LIST OF FIGURES ..... v
ACKNOWLEDGEMENTS ..... vii
ABSTRACT ..... viii
CHAPTER 1. INTRODUCTION ..... 1
1.1 Objective ..... 2
1.2 Organization of Thesis ..... 3
CHAPTER 2. REVIEW OF AREA ANGLE ..... 4
2.1 Calculation of area angle and power across the area ..... 4
2.2 Selection of Border Buses ..... 5
2.3 Evaluating the maximum power that can enter an area under single line contingencies ..... 6
2.4 Monitoring multiple outages ..... 8
2.4.1 Simple example ..... 8
2.4.2 Algorithm ..... 10
2.5 Exceptional outages ..... 12
CHAPTER 3. DESCRIPTION OF POWER SYSTEM NETWORK MODEL ..... 14
CHAPTER 4. AREAS, AREA ANGLES AND THRESHOLDS ..... 16
4.1 Idaho Area Simulation ..... 16
4.1.1 Outages and the calculation of thresholds for area angle ..... 16
4.2 Southern California Area ..... 19
4.2.1 Outages and area angle thresholds ..... 20
4.3 Robustness to changing stress direction ..... 22
4.4 Effect of exceptional outages ..... 24
CHAPTER 5. CONCLUSION ..... 26
REFERENCES ..... 28
APPENDIX. SYSTEM MODELLING AND COMPUTATION ..... 30

## LIST OF TABLES

## Page

Table 2.1 Simple example demonstrating that area angle tracks severity . . . . . . . . 10
Table 2.2 The possibilities of exceptional outages . . . . . . . . . . . . . . . . . . . . . 12
Table 2.3 Exceptional outages in simple network . . . . . . . . . . . . . . . . . . . . . 13

Table 3.1 Difference between the reduced model and detailed model . . . . . . . . . . 14

## LIST OF FIGURES

## Page

Figure 1.2 Border buses for a load area . . . . . . . . . . . . . . . . . . . . . . . . . . . 2
Figure 1.3 Border buses for a transfer path area . . . . . . . . . . . . . . . . . . . . . . 2

Figure 2.1 Area angle representation . . . . . . . . . . . . . . . . . . . . . . . . . . . . 5
Figure 2.2 Simple example on monitoring multiple outages . . . . . . . . . . . . . . . . 9
Figure 2.3 Simple network for illustrating exceptional outages . . . . . . . . . . . . . . 13

Figure 3.1 Angles at the buses plotted on WECC. The black color represents a higher value and blue color represents a lower value . . . . . . . . . . . . . . . . . . 15

Figure 4.1 Idaho Area - Border $M_{a}$ buses are colored in red and border $M_{b}$ buses area colored in blue. Numbers represent the weights . . . . . . . . . . . . . . . . 17
$\begin{array}{ll}\text { Figure } 4.2 & \text { Single line non-exceptional outages for Southern Idaho area. Outages are } \\ & \text { ordered so the maximum power increases. . . . . . . . . . . . . . . . . . . . } 17\end{array}$
Figure 4.3 Random sample of 1700 double line non-exceptional outages for Southern Idaho area18

Figure 4.4 Random sample of 1396 triple line non-exceptional outages for Southern
Idaho area ..... 18
Figure 4.5 Southern California Area ..... 19

Figure 4.6 Single line non-exceptional outages for Southern California area. Outages are ordered so that maximum power increases20

Figure 4.7 Random sample of 512 double line non-exceptional outages for Southern California area.21
Figure 4.8 Random sample of 2020 triple line non-exceptional outages for Southern California area.21
Figure 4.9 Single line outages for reduced BPA network. Pattern of stress used here is according to weights at the buses ..... 23
Figure 4.10 Single line outages for reduced BPA network. Pattern of stress used here is proportional to the tie line power flows into the buses. ..... 24
Figure 4.11 All single line outages for Idaho area with exceptional outages colored in red ..... 25
Figure 4.12 Random sample of triple line outages for Idaho area with exceptional outages colored in red ..... 25
Figure . 1 Modeling of three winding transformer ..... 32

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#### Abstract

Synchrophasors are especially useful for wide area monitoring, state estimation and various other applications. Hence, electric utilities are installing these synchrophasors in their areas. Power transfer through these areas stresses the system whenever there are outages inside the area. When state estimator fails to converge, fast indication of the amount of stress inside the area is necessary whenever there are multiple outages in the area. We use the synchrophasor angle measurements at the border buses of an area to calculate area angle which indicates the stress inside the area. The operator can use this indication to take necessary actions. The area angle is a scalar measure of power system area stress that responds to line outages within the area and is a weighted combination of synchrophasor measurements of voltage angles around the border of the area. The area angle has previously been tested in a transfer path areas on a reduced model. In this thesis, we further explore and test this concept on a detailed western interconnection model. Working on a detailed model has computational challenges. We explain techniques to achieve the computational efficiency. We also design new power system areas to extend the fast synchrophasor monitoring of multiple overloads with area angles. The new areas have loads in the middle of the area supplied by power flows into the area along its outer border. We examine setting actionable thresholds for curtailing area transfers when there are multiple outages and the robustness of the method to the pattern of stress.


## CHAPTER 1. INTRODUCTION

State estimation is used by power companies to observe the system and to take control actions when needed. Weighted least square methods are the most common algorithm used by the state estimator. The state estimator may not converge all the time because of errors in the topology of the network or large variations in initial values. from synchrophasor data gives the stress across the system when the power system state estimator does not converge.

The angle across an area of a power system is a weighted combination of synchrophasor measurements of voltage phasor angles around the border of the area (3;7). The weights are calculated from a DC load flow model of the area in such a way that the area angle satisfies circuit laws. Area angles were first developed for the special case of areas called cutset areas that extend all the way across the power system $(4 ; 5 ; 6)$. This was generalized to an area with power flow in one principal direction through the area as illustrated in Fig.1.3 and demonstrated with the north-south flow from Canada to California through an area that included Washington and Oregon states (10). There are also AC versions of area angles $(3 ; 15)$. The area angle describes the stress on the area due to the power transfer, and an increase in the angle corresponds to increasing the loading of the area lines carrying the power transfer. (Since the area angle satisfies circuit laws, the intuition for this behavior is similar to the simple special case of two parallel circuits joining two buses: If one of the circuits outages, the total power flow transfers to the remaining circuit, but the angle between the two buses increases because the remaining circuit has higher impedance than the double circuit.) The purpose of setting up a specific area stressed with a specific power transfer from one area border to another area border is so that the corresponding area angle across the area and between those borders can be a meaningful scalar measurement for which thresholds for emergency action can be set.


Figure 1.2 Border buses for a load area


Figure 1.3 Border buses for a transfer path area

For the area angle defined in (10), actionable thresholds for the area angle were developed so that the north-south power transfer could be quickly curtailed on an emergency basis if the area angle exceeded its threshold. This fast indication of the need for emergency action is particularly useful for multiple outages in which there is a possibility that the state estimator will not converge, so that the methods based on state estimation of detecting and mitigating line overloads are not available.

The new areas we introduce in this thesis consist of one boundary $M_{a}$ enclosing the entire area and another boundary $M_{b}$ formed by major loads in the middle of the area as shown in Figure 1.2. The power flow pattern of interest is the import of power into the area through boundary $M_{a}$ and its flow through the area to the loads $M_{b}$ inside. This pattern of power flow stresses the area by loading its lines, and the area angle between $M_{a}$ and $M_{b}$ measures this stress.

All the results are based on a 19402 bus, 2024 summer base case DC load flow model of the WECC system unless otherwise stated. Throughout the thesis, we do not address the line outages inside the area that island the area.

### 1.1 Objective

The angle difference between two buses indicate a stressed power system. But an area of a power system has several thousand buses and looking at the angle difference between any two buses does not give actionable information of the stress in the system. We reduce the area to a single line and
combine the synchrophasor measurements at the border buses of an area to form an area angle. The area angle is the angle difference between the two buses of the reduced network and gives the stress across the system. Thresholds can be set for the area angle to monitor multiple line outages inside the area.

The specific objectives of this thesis are the following:

1. Test the area angle for new types of areas
2. Check the robustness for different patterns of stress while setting up actionable thresholds.
3. Study the effects the detailed model has on the computation process and develop efficient algorithms for better computation.
4. Explore calculating the area angle and setting up the thresholds in a twenty thousand WECC bus system.

### 1.2 Organization of Thesis

The first chapter gives a brief introduction on the intuition behind area angle and the objectives of the thesis. Chapter 2 gives a detailed description of the calculation of area angles and setting up actionable thresholds for them for monitoring. Chapter 3 describes the detailed WECC model. Chapter 4 gives the simulation results for new areas, robustness to stress patterns, effects of exceptional outages and the effect of model detail. Chapter 5 concludes the thesis.

## CHAPTER 2. REVIEW OF AREA ANGLE

### 2.1 Calculation of area angle and power across the area

Area angle is a scalar quantity obtained from angle measurements from the Phasor Measurement Units (PMUs or synchrophasors) at the border buses of an area. These angle measurements from PMUs are combined in a linear combination with weights to give one area angle. (The detail description and calculation of the area angle is found in $(1 ; 7))$. This can be thought of as reducing the area to a single transmission line and finding the angle across the line which gives us the stress across it.

After choosing the area of interest, the border buses of an area must be identifed. The selection of border buses depends on the type of area and is described in section 2.2. The power entering into the area is the power flowing across the tie lines (tie lines which are outside the area), $P_{\text {tie }}^{m}$ and the power injections at the border buses $P^{m}$. All the buses inside the area are reduced by the Kron reduction process. The power injections across the interior buses are also reduced by Kron reduction and replaced as power injections at the border buses $P_{\text {int }}$. Hence the total power into the reduced area is the sum of internal and external powers:

$$
\begin{equation*}
P=P_{t i e}^{m}+P^{m}+P_{i n t} \tag{2.1}
\end{equation*}
$$

Now the reduced area consists of border buses and the lines connecting them as shown in figure 2.1. This network can be further reduced to a single transmission line. The power injections across its nodes $P_{a b}$ is the sum of power injections across the border buses. The susceptance of the reduced line depends on the susceptances of the lines inside the network. Thus the value of susceptances changes whenever there is a change inside the network (For example outages of the lines inside changes the configuration of the network and changes susceptance of the reduced line). The angles at the border buses $M_{a}$ have positive weights and angles at border buses $M_{b}$ have negative weights.


Figure 2.1: Area angle representation

We assume angle measurements from synchrophasors are available at the chosen border buses of an area. Area angle is computed as a weighted combination of angles across the border buses of an area. If the border buses are ordered as $1,2,3, \ldots, m$ and angles across these buses are $\theta_{1}, \theta_{2}, \ldots, \theta_{m}$ then the area angle is computed as

$$
\begin{align*}
\theta_{\text {area }} & =\sum_{j=1}^{m} w_{j} \theta_{j}  \tag{2.2}\\
w_{j} & =\frac{\sigma_{a} B_{\text {eq }}}{b_{\text {area }}}  \tag{2.3}\\
b_{\text {area }} & =\sigma_{a} B_{e q} \sigma_{a}^{T}  \tag{2.4}\\
\sum_{j=1}^{m} w_{j} & =0 \tag{2.5}
\end{align*}
$$

Here $\sigma_{a}$ is the row vector of length $m$ with entries of 1 at positions where there are $M_{a}$ buses and the rest are zero. $B_{e q}$ is the equivalent susceptance matrix of the border buses and $b_{a b}$ is the bulk susceptance of the area. The power flow across an area is then

$$
\begin{equation*}
P_{a b}=b_{a b} \theta_{\text {area }} \tag{2.6}
\end{equation*}
$$

### 2.2 Selection of Border Buses

The area angle calculation and maximum power that can enter the area depends on the susceptance of the area, which in turn depends on the configuration of the area chosen. Different
configurations of the area can be obtained by choosing a different set of border buses and for each of these configurations, area angle will be different since the susceptances change. For the area angle to be calculated, the border buses of an area must be chosen in such a way that isolating the border buses disconnects the area from the rest of the network. In other words, the tie lines connecting the border buses must form a cutset.

The border buses of an area can be categorized into two sets. The first set of border buses must be chosen such that power entering into those buses along the tie lines must be positive. The second set must be chosen such that power entering those buses is negative. For example consider the BPA area (Figure ??) which acts as a transmission corridor transferring power from northwest Canada to northern California. In this case northern border buses are named as $M_{a}$ border buses and southern border buses are named as $M_{b}$ border buses. For the load areas such as the southern California area, the power is entering across all its outer border buses. In this case, all the outer border buses are selected to be $M_{a}$ border buses and some load buses inside the area are selected to be $M_{b}$ border buses. The choice of the $M_{b}$ load buses depends on the availability of PMU measurements. Generally, electric utilities install PMUs at the largest load buses to monitor various parameters and hence we select these buses to be $M_{b}$ border buses.

### 2.3 Evaluating the maximum power that can enter an area under single line contingencies

Previous work uses area angle to define thresholds to monitor stress across the area under various contingencies (10) and showed that area angle is a good indicator of stress. The strategy is to define thresholds based on the line limits in terms of maximum power transfer through the area and convert the maximum power transfer threshold to an equivalent area angle threshold to monitor stress across the area. Then data from PMU are combined to measure the area angle and this area angle is compared to the threshold. If the measured area angle is greater than the threshold, then it is an indication that the system is under stress and action is required.

To evaluate the maximum power that can enter the area it is necessary to stress the area with additional power injections (10). These additional power injections can be made at border buses in proportion to the tie line flows entering or leaving at the border buses and in proportion to the net injection at the buses inside the area. That is, to find out the maximum power injection at the border buses we can inject the additional power at the border buses $M_{a}$ and $M_{b}$.

To calculate the additional power injections, we first apply any contingencies by removing lines inside the area and then the DC power flow is calculated and the power flows through all the lines is determined. We need to calculate the amount of power that needs to be injected at the border buses such that line $k$ reaches its limit in case of contingency $i$. The power injection is calculated from the generation shift factor as

$$
\begin{align*}
\Delta P^{a b(i) k m a x} & =\frac{\Delta P_{k}^{\operatorname{limit}(i)}}{\rho_{k}^{a b(i)}}  \tag{2.7}\\
\rho_{k}^{a b(i)} & =b_{k}\left(e_{u}^{T}-e_{v}^{T}\right)\left(\left(B^{i}\right)^{-1}\right)\left(e_{a}-e_{b}\right) \tag{2.8}
\end{align*}
$$

where $\Delta P_{k}^{\text {limit }(i)}$ is the difference between the power flow and its rated power flow in line $k$ after contingency $i . \rho_{k}^{a b(i)}$ is the generation shift factor of the line $k$ with respect to the power at border buses $M_{a}$. $B_{i}$ is the susceptance matrix after line $i$ is outaged, $b_{k}$ is the admittance of the line $i$. $u$ and $v$ are the sending and receiving end buses of line $k$. For $e_{a}$, we can choose its entries corresponding to the $M_{a}$ buses according to weights $\alpha_{j}, j \in M_{a}$ that are the fraction of power flows along the tie lines connecting the border buses:

$$
\begin{equation*}
\alpha_{j}=\frac{P_{\text {intoj }}}{P_{\text {intoa }}} \tag{2.9}
\end{equation*}
$$

Similarly for $e_{b}$, we can choose $\alpha_{j}=\frac{P_{\text {intoj }}}{P_{\text {intob }}}, j \in M_{b}$. Alternatively, the weights can be chosen as the weights $w_{j}$ used to define the area angle. These alternatives are discussed further in Chapter 4 of this thesis.

The maximum possible extra power injection into the border buses when line $i$ is out such that the system satisfies the $\mathrm{N}-1$ contingency criteria is the minimum of all additional power injection that satisfies all the line limits.

$$
\begin{equation*}
\Delta P^{i n j}=\operatorname{Min}\left\{\Delta P^{a b(1) k \max }, \Delta P^{a b(2) k \max }, \ldots \Delta P^{a b(n) k \max }\right\} \tag{2.10}
\end{equation*}
$$

If the power is injected at border buses $M_{a}$ then the share of power injection at each border bus is

$$
\begin{equation*}
\Delta P_{j}=\alpha_{j} \Delta P^{i n j} \tag{2.11}
\end{equation*}
$$

Then the maximum power that can enter the area at border buses $M_{a}$ is the sum of power across the cutset of lines joining the $M_{a}$ border buses to the network outside the area and additional power calculated from equation 2.10 and the power injections at border $M_{a}$ buses.

$$
\begin{equation*}
P_{m a}^{\max (i)}=\Delta P^{\text {injout }}+\sigma_{a} P^{\text {minto }(i)}+P_{m a} \tag{2.12}
\end{equation*}
$$

where $P^{\text {minto(i) }}$ is the sum of power flows through the $M_{a}$ border buses when line $i$ is outaged.
While calculating the area angle for different line outages, we do not consider the line outages which isolate the system when removed such as the radial distribution lines.

A threshold is set to be the area angle that occurs when the worst case single line outage happened. When area angle computed after other line outages is less than this area angle, then emergency measures should be taken to curtail the power transfer through the area.

### 2.4 Monitoring multiple outages

### 2.4.1 Simple example

Suppose the area is a transfer path area and power is transferred from north to south buses. Whenever a contingency happens, the power flow and voltage angles get redistributed. Whenever there is no path parallel to and outside the area, the tie flow into the area does not change. Whenever there is a high impedance path parallel to the area, the tie flow into the area does not change much. Hence, in these cases, the power entering the area is the same or approximately the same in both base case and in case of contingencies.

Consider the simple example of the case of three parallel lines connecting two buses as shown in Figure 2.2. The power entering the area is the same in all cases (base case and contingency cases).

Here we show in this simple example, the procedure to set up angle threshold to monitor multiple


Figure 2.2: Simple example on monitoring multiple outages

Consider the case where line 3 is out. The powers flowing in lines 1 and 2 are 93.75 MW and 56.25 MW respectively. The maximum power that can enter into the area depends on the line limits and power flows. The additional power that should be injected at the north bus such that line 2 reaches its limit is $90 \mathrm{MW}\left(\Delta P_{i n j} \frac{300}{800}=90-56.25\right)$. Therefore for line 2 to reach its limit, the maximum power that should be injected at the north bus is $P_{i n j}=150+90$. Similarly, for line 1 to reach its limit, the power that should be injected is $P_{i n j}=150+50$. Therefore, the maximum power that can enter into the area without violating the $\mathrm{N}-1$ contingency criteria is minimum of the above maximum powers which is 200 MW . The area angle in this case is $10.74^{\circ}$. We can compute the maximum power that can enter the area and the area angle for the cases when lines $1,2,1 \& 2$, $1 \& 3,2 \& 3$ each outage and these cases are given in table 2.1

Table 2.1: Simple example demonstrating that area angle tracks severity

|  | Base case |  |  | line 1 out |  |  | line 2 out |  |  | line 3 out |  |  | lines 1 \& 2 out |  |  | lines $1 \& 3$ out |  |  | lines 2 \& 3 out |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Lines | P | $\theta_{a b}$ | $P_{\text {max }}$ | P | $\theta_{a b}$ | $P_{\text {max }}$ | P | $\theta_{a b}$ | $P_{\text {max }}$ | P | $\theta_{a b}$ | $P_{\text {max }}$ | P | $\theta_{a b}$ | $P_{\text {max }}$ | P | $\theta_{a b}$ | $P_{\text {max }}$ | P | $\theta_{a b}$ | $P_{\text {max }}$ |
| 1 | 75 |  | 250 | 0 |  | - | 107.14 |  | 175 | 93.75 |  | 200 | 0 |  | - | 0 |  | - | 150 |  | -25 |
| 2 | 45 | $8.59^{\circ}$ | 300 | 90 | $17.18^{\circ}$ | 150 |  | $12.27^{\circ}$ | - | 56.25 | $10.74{ }^{\circ}$ | 240 | 0 | $42.97^{\circ}$ |  | 150 | $28.64^{\circ}$ | -60 | 0 | $17.18^{\circ}$ |  |
| 3 | 30 |  | 300 | 60 |  |  |  |  | 210 |  |  |  |  |  | -90 |  |  |  |  |  |  |
|  |  | $8.59^{\circ}$ | $\begin{aligned} & P_{\max }^{\min }= \\ & 250 \end{aligned}$ |  | $17.18^{\circ}$ | $\begin{aligned} & P_{\max }^{\min }= \\ & 150 \end{aligned}$ |  | $12.27^{\circ}$ | $\begin{aligned} & P_{\max }^{\min }= \\ & 175 \end{aligned}$ |  | $10.74{ }^{\circ}$ | $\begin{aligned} & P_{\max }^{\min }= \\ & 200 \end{aligned}$ |  | $42.97^{\circ}$ | $\begin{aligned} & P_{\max }^{\min }= \\ & -90 \end{aligned}$ |  | $28.64{ }^{\circ}$ | $\begin{aligned} & P_{\max }^{\min }= \\ & -60 \end{aligned}$ |  | $17.18^{\circ}$ | $\begin{aligned} & P_{\max }^{\min }= \\ & -25 \end{aligned}$ |

From table 2.1, we can see that in the case of single line outages, as the maximum power that can enter the area decreases from 250 MW to 150 MW , the area angle increases from $8.59^{\circ}$ to $17.18^{\circ}$. Similarly, for double line outages, the maximum power that can enter the area decreases from -25 MW to -90 MW , thus effectively violating the N-1 contingency criteria. The area angle in this case increases from $17.18^{\circ}$ to $42.97^{\circ}$.

The area (Figure 2.2) is designed such that it follows N-1 criteria. Now, we set the maximum power entering the area in the worst case single contingency to be the emergency power threshold. Since the maximum power is a hypothetical quantity and cannot be monitored, we convert the maximum power emergency threshold to emergency area angle threshold. The area angle can be computed from the measurements of synchrophasor data available at border buses.

Following the procedure, we set the emergency power threshold in the simple example (2.2) to be 150 MW i.e., the worst case single contingency happens when line 1 is out and the corresponding area angle is $17.18^{\circ}$. We choose this area angle to be emergency area angle threshold. We can see that, in case of multiple outages, the area angle is always above the emergency area angle threshold, thus effectively indicating that multiple outages inside the area are worse than the worst case N-1 case, and require emergency action to reduce the transfer through the area.

### 2.4.2 Algorithm

We now summarize the area angle algorithm from (10).

## Offline calculations

1. Choose the border buses of an area such that the border buses form a cutset (section 2.2).
2. For each single outage inside the area, after the outage, calculate the maximum power that can enter the area before the first line limit is encountered. The maximum power that can enter the area for the worst case single outage is the emergency threshold for the maximum power entering the area. Also define the alarm threshold on the maximum power entering the area.
3. Set the base case power entering the area to the emergency threshold of the maximum power. Then for all single outages inside the area, calculate the area angle after the outage.
4. After eliminating the exceptional outages, choose the minimum of all the maximum powers from step 2 as the emergency power threshold (This is because this is the maximum amount of power that can enter into the area under worst case single contingency without violating N -1 contingency criteria). The corresponding area angle will be emergency area angle threshold.

## Online implementation

1. Continuously observe the system using synchrophasors at the border buses. Get the values of voltage angles from the synchrophasors in real time.
2. Calculate the area angle using the weights computed offline.
3. If outages which are causing local power redistribution problems have not occurred, then compare the area angle to its thresholds to take no action or to take proper action with the appropriate urgency.
4. If the area angle is above the emergency area angle threshold (it implies that the multiple outages are worse than worst case single line contingency and not following the $\mathrm{N}-1$ criteria), then take emergency actions to reduce the power in the tie lines of the area.

### 2.5 Exceptional outages

It is not always the case that the area angle is inversely related to the maximum power that can enter the area. In our experience we encountered some cases in which there are exceptional outages for which the area angle does not necessarily track the maximum power that can enter into the area. In (10), Darvishi explained one illustrative example of an exceptional outage where the area angle increases but the maximum power entering the area remains the same for the simple network of Figure 2.3. Here we show additional cases by changing the susceptance and line limit values for the same illustrative example.

The various possibilities of exceptional outages are given in table 2.2. In order to explain

Table 2.2: The possibilities of exceptional outages

| $\boldsymbol{P}_{\text {into }}^{\max }$ | $\boldsymbol{\theta}_{\boldsymbol{a b}}$ |
| :--- | :--- |
| Decreases | Constant |
| Increases | Constant |
| Constant | Decreases |
| Constant | Increases |
| Decreases | Decreases |
| Increases | Increases |
| Constant | Constant |

all the possibilities of exceptional outages in table 2.2, consider the following network in Figure 2.3. The line limits are chosen such that the network satisfies the N-1 criteria. In this network the power entering into the area (bus 1) is 100 pu and is always the same in base case and in case of contingencies. However, the maximum power that can enter into the area changes. The power out is 80 pu and load is 20 pu in all the cases. By changing the limits and susceptances of the lines inside the area, we can demonstrate all the possible exceptional cases as shown in table 2.3. The single line outages which are not following the inverse relation are exceptional outages. This more general analysis of exceptional outages should be useful in understanding cases in which the area angle does not work as expected.


Figure 2.3: Simple network for illustrating exceptional outages

Table 2.3: Exceptional outages in simple network

| $P_{\text {limit } 12}$ | $b_{12}$ | $P_{\text {limit } 13}$ | $b_{13}$ | $P_{\text {limit } 23}$ | $b_{23}$ | line 12 out |  | line 13 out |  | $P_{\text {into }}^{\max }$ | $\theta_{13}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | $P_{\text {into }}^{\max }$ | $\theta_{13}$ | $P_{\text {into }}^{\max }$ | $\theta_{13}$ |  |  |
| 100 | 40 | 100 | 30 | 80 | 20 | 100 | 3.3 | 100 | 6.5 | constant | increases |
| 100 | 50 | 120 | 30 | 80 | 60 | 120 | 3.3 | 100 | 3.3 | decreases | constant |
| 100 | 50 | 120 | 20 | 80 | 60 | 120 | 5 | 100 | 3.3 | decreases | decreases |
| 100 | 50 | 100 | 20 | 80 | 60 | 100 | 5 | 100 | 3.3 | constant | decreases |
| 120 | 50 | 100 | 30 | 100 | 60 | 100 | 3.3 | 120 | 3.3 | increases | constant |
| 120 | 40 | 100 | 30 | 100 | 20 | 100 | 3.3 | 120 | 6.5 | increases | increases |
| 100 | 50 | 100 | 30 | 80 | 60 | 100 | 3.3 | 100 | 3.3 | constant | constant |

## CHAPTER 3. DESCRIPTION OF POWER SYSTEM NETWORK MODEL

The data used for modeling and simulation is the North American Western interconnection (WECC) system. The details and the difference between reduced and detailed model are given in table 3.1. The network formed is given in Figure 3.1. The top part of the graph corresponds

Table 3.1: Difference between the reduced model and detailed model

|  | Detailed model | Reduced model |
| :--- | :---: | :---: |
| Number of buses | 20194 | 1553 |
| Number of loads | 10951 | 898 |
| Number of transmission lines | 16912 | 2115 |
| Number of generators | 4254 | 493 |
| Number of two winding transformers | 7654 | 243 |
| Number of three winding transformers | 561 | 0 |
| Number of HVDC lines | 3 | 0 |
| Number of areas | 239 | 64 |

to those buses which are in Canada followed by those buses which are in Washington, Montana, Oregon, Idaho, Nevada, Utah, Colorado and the bottom part of graph corresponds to buses which are in southern California, Arizona. The DC load flow voltage angle is computed for all the buses in the network and these are shown by the colors in Figure 3.1. It can be seen that angle is higher at northern buses and tends to reduce as one progresses towards the south, indicating that power is transferred from northern part of Washington state (where there is generation available) to load parts such as south western part of California (where there are major load centers) with some power consumed in between. In this thesis, we explain the area angle concepts on Southern California region and Southern Idaho region that are essentially load areas, and the BPA area which covers the states of Washington, Oregon and part of Idaho. The BPA area acts as transmission corridor transferring power from Canada to California.


Figure 3.1: Angles at the buses plotted on WECC. The black color represents a higher value and blue color represents a lower value


## CHAPTER 4. AREAS, AREA ANGLES AND THRESHOLDS

We calculated the area angle and set the thresholds on area angles for three different areas. Two areas in Idaho and in Southern California are load areas. The third area is a transmission corridor through the states of Washington and Oregon supplying power from Canada down to California.

### 4.1 Idaho Area Simulation

The Idaho area is a load area, where the power is consumed inside the area. According to the model, it has about 271 buses and 312 transmission lines. The Idaho area is given below with the $M_{a}$ border buses colored in red and $M_{b}$ border buses colored in blue and all the other buses inside the area are colored in black. The numbers across the border buses represents the weights at those buses.

### 4.1.1 Outages and the calculation of thresholds for area angle

All the lines inside the area are simulated to be outaged one at a time and after removing the exceptional outages, the results are plotted in 4.2.The minimum of the maximum powers that can enter the area is 33.2 pu with corresponding area angle 9.93 deg. So, the emergency threshold in this case will be 9.93 deg. (We do not set any alarm thresholds in this case as the slope of the maximum power is not clear.) The double and triple outages are given in figures 4.3 and 4.4 respectively after excluding the exceptional outages. We can see that the area angle emergency threshold discriminates the double and triple outages that have powers less than the emergency power threshold.


Figure 4.1: Idaho Area - Border $M_{a}$ buses are colored in red and border $M_{b}$ buses area colored in blue. Numbers represent the weights


Figure 4.2: Single line non-exceptional outages for Southern Idaho area. Outages are ordered so


Figure 4.3: Random sample of 1700 double line non-exceptional outages for Southern Idaho area


Figure 4.4: Random sample of 1396 triple line non-exceptional outages for Southern Idaho area.

### 4.2 Southern California Area

The Southern California is a load area with power transferred through an outer border and consumed inside the area. The Southern California area network buses and transmission lines are shown in Figure 4.5.


Figure 4.5: Southern California Area

The Southern California area consists of Los Angeles and regions surrounding Los Angeles and is shown by the darker buses and lines in Figure 4.5. The Southern California area network has 418 buses and 501 lines. Red buses in Figure 4.5 represent the outer border buses and blue buses are major loads in Los Angeles. Here we are assuming synchrophasor measurements are available at the specified buses. As of 2010, synchrophasors were installed or proposed at all the border buses except for one of the load buses. The trend is for synchrophasor measurements to become more widespread, particularly as modern relays can include synchrophasor measurements. We note
in computing the area angle that border buses with very small weights can be omitted from the calculation so that synchrophasor measurement at those buses is not required.

### 4.2.1 Outages and area angle thresholds

The single line outages are given below.


Figure 4.6: Single line non-exceptional outages for Southern California area. Outages are ordered so that maximum power increases

The power entering the area during the worst case single contingency is approximately 93 pu and the corresponding area angle is 58 pu (approximately). The double and triple outages are given below with the emergency thresholds for area angle and max power and it can be seen that the thresholds clearly detect the double and triple outages which are worse than worst case single contingency.


Figure 4.7: Random sample of 512 double line non-exceptional outages for Southern California area.


Figure 4.8: Random sample of 2020 triple line non-exceptional outages for Southern California area.

### 4.3 Robustness to changing stress direction

When determining the area angle thresholds, it is necessary to choose a pattern of stress by which the power transfer is increased. When a line is out, the participation factors for all the remaining lines are calculated with respect to power injections at the border buses. The maximum power that can be injected across the border buses such that line $k$ inside the area reaches its limit is obtained by dividing the limit of that line by its participation factor according to equation (4) in (10). Then the amount of power to be injected into the border buses is the minimum of injections across all the lines. The amount of power to be injected at the border bus such that line $k$ reaches its maximum power limit depends on the generation shift factor of line $k$. According to equation (8) in (10) the generation shift factor for line $k$ with respect to injections at border buses is given by

$$
\rho_{k}^{a b(i)}=b_{k}\left(e_{u}^{T}-e_{v}^{T}\right)\left(\left(B^{i}\right)^{-1}\right)\left(e_{a}-e_{b}\right)
$$

Here $e_{a}$ and $e_{b}$ are $n \times 1$ vectors ( $n$ is the number of buses) with values only at the positions of border buses and the rest are zero. We choose the values in these column vectors to specify the pattern of stress. The first method is to choose $e_{a}$ and $e_{b}$ according to the weights at the buses. The second method chooses $e_{a}$ and $e_{b}$ according to $\alpha=P_{\text {intoj }} / P_{\text {intom }}$. The pattern of stress is defined by how the border buses participate in changes in the power transfer.

We compare these two different methods of stressing each area to assess the robustness of the thresholds when the power stress pattern is changed.

1. The first method increases the power at border bus $i$ proportional to the power injected at border bus $i$ from outside the area.
2. The second method increases the power at border bus number $i$ proportional to the area angle weight $w_{i}$.


Figure 4.9: Single line outages for reduced BPA network. Pattern of stress used here is according to weights at the buses

Here we compare the single line outages of BPA network in a reduced network. Single line outages for the reduced BPA area from (10) is reproduced here for easy reference in Figure 4.9. The outages are ordered according to increasing amount of maximum power transfer into the area. The results in (10) for the north-south transfer of power through the BPA area are obtained with the second method, so here we give the results for the first method ${ }^{1}$. The weights at the border buses used for stressing the system in (10) are

$$
\begin{array}{r}
w=\{0.23,0.0057,0.0078,0.1,0.18,0.017,0.6, \\
-0.03,-0.4,-0.38,-0.18,-0.004\} \tag{4.1}
\end{array}
$$

The weights at the border buses which are proportional to tie line flows are

$$
\begin{array}{r}
\alpha=\{0.68,0.0111,0.002,0.056,0.019,0.042,0.18, \\
-0.002,-0.4,-0.35,-0.23,-0.029\} \tag{4.2}
\end{array}
$$

We note the substantial difference in the weights in (4.1) and (4.2).
The single outages of the system stressed according to $\alpha$ are given in Figure 4.10. Comparing Figure 4.10 with Figure 4.9 we can see that the results are similar and the thresholds do not depend on the method of stress.

[^0] U


Figure 4.10: Single line outages for reduced BPA network. Pattern of stress used here is proportional to the tie line power flows into the buses.

### 4.4 Effect of exceptional outages

The formulation of the areas is such that power enters into the area through one set of border buses and transfers or get consumed at the other set of border buses. The radial lines inside the area do not participate in the power transfer. The meshed lines in the area can participate in the power transfer. There is a general tendency for an outage of one the meshed lines to decrease the maximum power that can be transferred through the area, since the outage tends to transfer more power to the parallel paths. Moreover, the outage tends to increase the impedance across the area while the power transfer remains constant. Since the area angle obeys circuit laws, it follows that the area angle generally increases. Thus there is generally an inverse relationship between area angle and maximum power transfer that we exploit in the monitoring. However, there are some exceptional cases in which the area angle does not respond inversely to the maximum power transfer. These exceptional cases usually correspond to local power redistribution problems as explained in (10) and further in Chapter 3 of this thesis. Depending on the nature of the lack of response, some of the lines giving exceptional outages need to be independently monitored to improve the accuracy of the interpretation of the area angle changes. Fig. 4.11 shows all the single
line outages for the Idaho area, including the exceptional outages shown as red dots. Fig. 4.12 shows all the triple line outages for the Idaho area that include at least one exceptional outage.


Figure 4.11: All single line outages for Idaho area with exceptional outages colored in red


Figure 4.12: Random sample of triple line outages for Idaho area with exceptional outages colored in red

## CHAPTER 5. CONCLUSION

Our overall philosophy to extract actionable information from synchrophasor data is to restrict the problem to one pattern of stress for one phenomenon, and suitably combine data from multiple synchrophasors into a meaningful scalar to be monitored. This restriction of the problem then allows not only detection of emergency conditions when the scalar exceeds a threshold, but specific mitigation actions to correct the problem. For example, we use an area angle to monitor a specific power transfer through a specific area with respect to thermal limits in (10) and monitor with a scalar index a specific transmission corridor with respect to voltage collapse in (15).

To advance and make workable this philosophy for area angles monitoring thermal limits inside an area, it is necessary to develop the art of choosing areas and power transfers stressing these areas, and show that meaningful thresholds can be developed. In this thesis, we extend the area angle monitoring to a different sort of area in which the power flows from the border of the area into large loads well inside the area. The emergency thresholds for area angle developed in relation to the $\mathrm{N}-1$ criterion seem effective in discriminating the triple outages that require no quick action from those that require quick emergency action to curtail the power transfer. In the case of BPA area which acts as transfer path area, the initial results suggest that thresholds cannot be set up.

We also briefly assess the performance of the area angle with respect to power stress direction. We have a choice of stressing the system either according to the weights or according the proportional tie line power flows. The maximum power that can enter the area remained the same irrespective of the type of the stress and the emergency thresholds that are setup discriminates the multiple outages.

The time required to run the offline algorithm of calculating and setting up the thresholds will be in the magnitude of days if computed by brute force methods. The techniques introduced () helped achieve computational efficiency. Even after employing these techniques, the time required
for offline algorithm varies from hours to days depending on the size of the area. Online algorithm calculations are fast since these depend on the voltage angle measurements from the synchrophasors at the border buses of the area.

This thesis explored setting up thresholds for transfer path areas and load areas. For future work, this method should be tested on the generation area. In case of load areas, the loads are assumed to be constant. The effect of dynamic loads on the area angle and thresholds can be studied. The method is far from perfect in terms of computational time for offline calculations. more efficient computation can be achieved by utilizing the inbuilt functions of software in which the algorithms are written.

Our testing of area angle extends the capability to combine synchrophasor measurements into a scalar index to monitor a specific problem and determine if emergency action needs to be taken or not. This fast action based on synchrophasor measurements is particularly useful for multiple contingencies when it is possible that the state estimator may not converge.

I gratefully acknowledge access to the WECC power flow data that enabled this research. The analysis and conclusions are strictly those of the author and not of PSerc, WECC, DOE, or SCE.

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## APPENDIX. SYSTEM MODELLING AND COMPUTATION

As described in the previous chapters, all the simulations are run on a twenty thousand bus Western Interconnection (WECC) system and all the calculations are done using Mathematica software. This chapter describes the modeling and steps to improve the computational efficiency. The data available for simulation is in the PSLF epc format. We have the choice of running DC load flow either in PSLF or PSSE software and export the necessary details for further calculations in any format supported by Mathematica. Since we have the framework set up to read PSSE .raw file format in Mathematica, we exported the .epc file as a .raw file.

A detailed system is always challenging in terms of modeling, computational efficiency, data analysis etc. Techniques are employed to reduce the computational time and memory required for computations. In the offline calculations testing area angle and setting thresholds, line outages have to be simulated. Whenever a line is out, the network topology changes and the susceptance matrix changes. In the calculation of participation factor, inverse of susceptance matrix is used as shown in equation 2.8. The network has approximately 20000 buses, so the susceptance of network is a matrix with 20000 rows and 20000 columns. Inverse calculation of such a matrix involves huge computational time and memory. We use the matrix inversion lemma to calculate the inverse of such a huge matrix and this is further explained in section . We achieved better computational time efficiency by utilizing many of the inbuilt functions in Mathematica. The computational time and memory required to simulate the area depends on the size of the area.

## Branches

The DC power flow is given by the equation

$$
\begin{equation*}
P=B \theta \tag{.1}
\end{equation*}
$$

where $P_{n \times 1}$ is the column vector of power injections at the $n$ buses, $\theta_{n \times 1}$ is a column vector of voltage angle at the buses, and $B_{n \times n}$ is the admittance matrix where

$$
\begin{align*}
B_{i i} & =-\sum_{n=1}^{k} b_{i n}  \tag{.2}\\
B_{i k} & =-b_{i k} \tag{.3}
\end{align*}
$$

and $b_{i k}$ is the susceptance of the line joining bus $i$ and bus $k$. In the DC load flow, all the branches are considered lossless. Therefore all the two winding transformers and transmission lines are represented with series impedance $z=r+j x, b=\frac{-1}{x}$.

## Three winding transformers

A three winding transformer modeling is different from a two winding transformer. While modeling a three winding transformer, we have the choice of a three bus equivalent model or a four bus equivalent model as shown in Figure .1. In the three bus model, the three winding transformer is split into two transformers. It does not consider the impedance of primary winding. In the four bus equivalent, it is split into three two-winding transformers. For the DC load flow calculation, the primary impedance can be ignored. Therefore, for the DC load flow calculations, the three bus equivalent is used to model the three winding transformer.

## HVDC lines

In the western interconnection, there are three HVDC connections

1. Eastern Alberta Transmission line, which is 485 km long interconnecting Newell HVDC static inverter plant near Brooks, Alberta with Heathfield static inverter plant near Gibbons, Alberta.
2. Pacific DC Intertie, also called Path 65, is 846 miles long interconnecting Celilo converter station in Oregon to Sylmar converter station north of Los Angeles.


Figure .1: Modeling of three winding transformer
3. Intermountain HVDC power line, also called Path 27, is 488 miles long interconnecting Adelanto converter station in Adelanto, California to Intermountain converter station in Delta, Utah.

In order to model the HVDC lines, we remove power from those buses where HVDC power is injected and add power to those buses where HVDC power is extracted. In other words, if $P_{d c}$ is the amount of power that is transferred through the HVDC line from bus $a$ to bus $b$, then at bus $a$, the injection will be $-P_{d c}$ and at bus $b$, the injection will be $P_{d c}$.

## Participation factor calculation

The area angle computation and the setting of area angle threshold helps us discriminate the double and triple line outages which are comparable to worst case single line outage. The shift factor of each line inside the area with respect to the border buses of the area is given by equation (2.8), which is copied here as

$$
\begin{equation*}
\rho_{k}^{a b(i)}=b_{k}\left(e_{u}^{T}-e_{v}^{T}\right)\left(\left(B^{i}\right)^{-1}\right)\left(e_{a}-e_{b}\right) \tag{.4}
\end{equation*}
$$

Here $e_{a}$ and $e_{b}$ have nonzero entries corresponding to buses $M_{a}$ and $M_{b} . e_{u}$ and $e_{v}$ have entry 1 at the starting bus position and ending bus position of line $k$ respectively.

After choosing a particular area, the values and positions of elements in the column matrices $e_{a}$ and $e_{b}$ do not change since they are ratios of base power injections at the border buses. The values of row matrices $e_{u}^{T}$ and $e_{v}^{T}$ do not change but the positions change according to the starting and ending buses of line $k$. However, these positions are within range of buses in the area since the area angle is computed only for the chosen area and all the values at the positions outside the range of area buses are zero. The structure of the matrices is given below

$$
\begin{gathered}
e_{u}^{T}-e_{v}^{T}=\left[\begin{array}{lllllll}
0 & 0 & \cdots & 0 & v_{1} & v_{2} & \cdots \\
v_{k} & 0 & \cdots & 0 & 0
\end{array}\right] \\
\left(\left(B^{i}\right)^{-1}\right)\left(e_{a}-e_{b}\right)=\left[\begin{array}{c}
x_{1} \\
x_{2} \\
\vdots \\
x_{n}
\end{array}\right] \\
\left(e_{u}^{T}-e_{v}^{T}\right)\left(\left(B^{i}\right)^{-1}\right)\left(e_{a}-e_{b}\right)=0 * x_{1}+0 * x_{2}+\cdots+v_{1} * x_{b 1}+v_{2} * x_{b 2}+\cdots+v_{k} * x_{b k}+\cdots+0 * x_{n}
\end{gathered}
$$

From the above equations, we can see that in the participation factor calculation, only the elements of the Binverse matrix whose positions are within the range of area buses are required. Hence we can form a sparse Binverse matrix which has elements only in the range of buses of the chosen area
and all other elements allowed to be zero. This not only reduces the memory to store the Binverse matrix but also increases efficiency in calculation of participation factor.

## $B$ inverse matrix calculation

To calculate the participation factor during single, double, and triple outages, it is necessary to calculate $B_{\text {out }}^{-1}$ matrix for each outage $i$. We calculate the $B_{\text {out }}^{-1}$ matrix from the matrix inverse lemma.

$$
\begin{gather*}
e_{i j}=\left[\begin{array}{c}
0 \\
\vdots \\
1 \\
\vdots \\
-1 \\
\vdots \\
0
\end{array}\right] \\
B_{\text {out }}=B+b_{i j} e_{i j} e_{i j}^{T} \tag{.5}
\end{gather*}
$$

Here $B_{\text {out }}$ is the $B$ matrix after line $k$ is out. $e_{i j}$ has 1 at the starting bus position of line $k$ and -1 at the ending bus position. $b_{i j}$ is the susceptance of the line $k$ with starting and ending bus positions at $i$ and $j$ respectively.

From the matrix inversion lemma

$$
\begin{equation*}
\left(B+c d^{T}\right)^{-1}=B^{-1}-B^{-1} c\left(I^{M}+d^{T} B^{-1} c\right)^{-1} d^{T} B^{-1} \tag{.6}
\end{equation*}
$$

where B is an $n \times n$ matrix, $c$ and $d$ are $n \times m$ matrices and $I$ is an $m \times m$ Identity matrix.
Applying the Matrix inversion lemma to our problem results in the calculation of $B_{\text {out }}^{-1}$ from base case $B^{-1}$ matrix:

$$
\begin{equation*}
B_{o u t}^{-1}=B^{-1}-\frac{B^{-1} b_{i j} e_{i j} e_{i j}^{T} B^{-1}}{1+b_{i j} e_{i j}^{T} B^{-1} e_{i j}} \tag{.7}
\end{equation*}
$$

Here $B^{-1}$ is the base case pseudoinverse of the $B$ matrix with elements only in the positions which are in the range of area buses.


[^0]:    ${ }^{1}$ The first method is described by equation (8) of (10). However the results presented in (10) tacitly use the

